Extracting the square root of twice (8),

$$2(x+y+z)=\pm\sqrt{\{2(a^2+b^2+c^2)\pm2\sqrt{[3(a^2+b^2+c^2)^2-6(a^4+b^4+c^4)]}\}}\,..\,(9).$$

Dividing (5), (6), and (7) in succession by (9),

$$x=\frac{b^2+c^2-a^2\pm\frac{1}{3}\sqrt{[3(a^2+b^2+c^2)^2-(a^4+b^4+c^4)]}}{\pm\sqrt{\{2(a^2+b^2+c^2)\pm2\sqrt{[3(a^2+b^2+c^2)^2-(a^4+b^4+c^4)]}\}}},$$

$$y=\frac{a^2+c^2-b^2\pm\frac{1}{3}\sqrt{[3(a^2+b^2+c^2)^2-(a^4+b^4+c^4)]}}{\pm\sqrt{\{2(a^2+b^2+c^2)\pm2\sqrt{[3(a^2+b^2+c^2)^2-(a^4+b^4+c^4)]}\}}},$$

$$z=\frac{a^2+b^2-c^2\pm\frac{1}{3}\sqrt{[3(a^2+b^2+c^2)^2-(a^4+b^4+c^4)]}}{\pm\sqrt{\{2(a^2+b^2+c^2)\pm2\sqrt{[3(a^2+b^2+c^2)^2-(a^4+b^4+c^4)]}\}}}.$$

Also solved by A. H. Holmes, J. Scheffer, and V. M. Spunar.

For a number of different solutions of this problem, when the known quantities are not squared, see *The Mathematical Magazine*, published by Dr. Artemas Martin, Vol. II, pp. 141-144, and pp. 193-196. ED. F.

GEOMETRY.

375. Proposed by C. N. SCHMALL, New York City.

From a point P on a circle there are drawn three chords PA, PB, PC. Show that the circles described on these chords as diameters intersect again in three collinear points.

I. Solution by S. G. BARTON, Ph. D., Clarkson School of Technology.

Take the point P as the origin of polar coördinates, and the diameter through P as the initial line. The coördinates of the points A, B, and C are, respectively, $2a\cos\alpha$, α; $2a\cos\beta$, β; $2a\cos\gamma$, γ; α, β, and γ being the vectorial angles.

The equations of the circles described upon the chords as diameters will be

$$\rho=2a\cos\alpha\cos(\theta-\alpha),$$
$$\rho=2a\cos\beta\cos(\theta-\beta),$$
$$\rho=2a\cos\gamma\cos(\theta-\gamma),$$

whence the coördinates of the points of intersection are

$$2a\cos\beta\cos\gamma,\ \beta+\gamma;\quad 2a\cos\gamma\cos\alpha,\ \gamma+\alpha;\quad 2a\cos\alpha\cos\beta,\ \alpha+\beta.$$

These points are all on the straight line whose equation is

$$2a\cos\alpha\cos\beta\cos\gamma=\rho\cos(\theta-\alpha-\beta-\gamma).$$

Join the points A, B, and C to make a triangle. The points of intersection are then the feet of the perpendiculars let fall from P upon the three sides, and the line through the points of intersection is the pedal line of P with respect to the triangle.

II. **Solution by S. LEFSEHETZ, Clark University.**

If we transform by inversion, the pole of inversion being in P, the transformed of the three circles of diameters PA, PB, and PC are perpendiculars at PA, PB, and PC in A', B', and C', points where these three lines meet the line obtained by transformation of the given circle. These three perpendiculars envelop a parabola of focus P; therefore, the circle circumscribed to the triangle they form passes through P, — a well known property of the parabola. By transforming back, we obtain a straight line and the proposition is thus proved.

Also solved by the Proposer.

PROBLEMS FOR SOLUTION.

ALGEBRA.

355. **Proposed by V. M. SPUNAR, M. and E. E., Chicago, Ill.**

Solve the equations:

$$\frac{1}{x_1}+\frac{1}{x_2}+\frac{1}{x_3}=a_1;$$
$$\frac{1}{x_2}+\frac{1}{x_3}+\frac{1}{x_4}=a_2;$$
$$\frac{1}{x_3}+\frac{1}{x_4}+\frac{1}{x_5}=a_3;$$
$$\cdots\cdots\cdots$$
$$\frac{1}{x_{n-2}}+\frac{1}{x_{n-1}}+\frac{1}{x_n}=a_{n-2};$$
$$\frac{1}{x_{n-1}}+\frac{1}{x_n}+\frac{1}{x_1}=a_{n-1};$$
$$\frac{1}{x_n}+\frac{1}{x_1}+\frac{1}{x_2}=a_n.$$

356. **Proposed by ARTEMAS MARTIN, Ph. D., Washington, D. C.**

Solve by quadratics, if possible, the equations,

$$w(x+y+z)=a,\quad x(w+y+z)=b,$$
$$y(w+x+z)=c,\quad z(w+x+y)=d.$$

[From the *Mathematical Magazine*, Vol. II, p. 256.]